

Turbulent Structure of the Interstellar Medium

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Summary. How does turbulence contribute to the formation and structure of the dense interstellar medium (ISM)? Molecular clouds are dense, high-pressure objects. It is usually argued that gravitational confinement causes the high pressures, and that the clouds are (magneto)hydrostatic objects supported by a balance between magnetic and turbulent pressures and gravity. However, magnetic pressures appear too weak, and MHD turbulent support not only requires driving, but also results in continuing gravitational collapse, as has now been demonstrated in simulations reaching 512^3 zones. Models of supernova-driven, magnetized turbulence readily form transient, high-pressure, dense regions that may form molecular clouds. They are contained not by self-gravity, but by turbulent ram pressures from the larger flow. Apparent virialization may actually be a geometrical effect. Turbulent clouds are unlikely to be in hydrostatic equilibrium, instead either collapsing or expanding, although they may appear well-fit by projected equilibrium Bonnor-Ebert spheres. Collapsing clouds probably form stars efficiently, while expanding ones can still form stars by turbulent compression, but rather inefficiently.

1 Equilibrium

The very high Reynolds numbers of the interstellar medium (ISM) ensure that it is turbulent and not laminar. How can we model a turbulent flow that is intrinsically non-steady and chaotic? Most models of the structure of the ISM rely on some notion of equilibrium: (magneto)hydrostatic equilibrium, virial equilibrium, energy equipartition, or at least statistical equilibrium (or stationarity). In this talk I use the structure of the dense ISM as a case study in the problems with these different assumptions. In this case, making such assumptions leads to the conclusion that molecular clouds are gravitationally bound, which may not generally be the case.

2 Standard Argument for Gravitationally Bound Clouds

The standard argument assumes that observed clouds are in virial equilibrium [24, 23, 21]. This was justified by apparent lifetimes estimated at 30 Myr by Blitz & Shu [8], which is many times the free-fall time τ_{ff} , and the seeming consistency of virial and actual mass given size-mass and size-linewidth

relations. Cloud parameters can then be estimated from the assumption of virial equilibrium, and (magneto)hydrostatic models constructed by detailed balance of gravity against turbulent and magnetic pressure support.

The full Eulerian virial theorem [22] for a volume of space is a time-dependent equation for the moment of inertia of the mass in the volume

$$(1/2)\ddot{I} = 2(\mathcal{E}_{th} + \mathcal{E}_{kin} - \mathcal{T}_{th} - \mathcal{T}_{kin}) + \mathcal{M} + \mathcal{W} - (1/2)\dot{\Psi}, \quad (1)$$

where \mathcal{E}_{th} , \mathcal{E}_{kin} , \mathcal{M} , and \mathcal{W} are thermal, kinetic, magnetic and gravitational energy, \mathcal{T}_{th} and \mathcal{T}_{kin} are thermal and ram pressure on the surface of the volume, and $\dot{\Psi}$ is the change in flux of the moment of inertia through the region.

This equation is usually simplified by neglecting the time-dependent and surface terms, and assuming clouds with homogeneous density and pressure. Below I will address how well justified these assumptions are, but if they are made, the equation can be recast to give external pressure [21]

$$P_{ext} = \frac{1}{4\pi} \left(-\alpha \frac{GM^2}{R^4} + \beta \frac{\Phi^2}{R^4} + 3 \frac{c_s^2 M}{R^3} + \frac{\sigma^2 M}{R^3} \right), \quad (2)$$

where M , R , σ , and Φ are the mass, radius, velocity dispersion, and enclosed magnetic flux of a cloud, G is the gravitational constant, c_s is the sound speed, and α and β are geometrical constants of order unity. If gravity and magnetic pressure are ignored, we recover the generalized Boyle's law $P_{ext}V = (c_s^2 + \sigma^2)M$. If, on the other hand, external, thermal, and magnetic pressure are all ignored, the equilibrium is reduced to a balance between gravity and turbulence:

$$\alpha GM/R = \sigma^2 \Rightarrow M_{vir} = \sigma^2 R/(\alpha G), \quad (3)$$

which gives the usual definition for the ‘‘virial mass’’ M_{vir} . This is certainly appropriate for isolated stellar clusters, where all the other terms are indeed negligible, but may be rather more problematic for gas clouds embedded in a turbulent flow.

Clouds will rarely or never have exact equality between gravity and internal pressure (thermal, magnetic, and turbulent). McKee [21] writes the *time-averaged* virial theorem in terms of the mean internal pressure $\bar{P} = P_{ext} - \mathcal{W}(1 - \mathcal{M}/|\mathcal{W}|)/3V$. The total energy of the cloud

$$E = (3V/2)[P_{ext} + \mathcal{W}(1 - \mathcal{M}/|\mathcal{W}|)/3V]. \quad (4)$$

If magnetic fields are unimportant, the cloud is gravitationally bound if $P_{ext}V < -\mathcal{W}/3$, that is if the mean weight in virial equilibrium exceeds the surface pressure.

If we assume the ISM to be in pressure equilibrium, then we can take the ISM pressure to be the external pressure. Boulares & Cox [9] argue for an ISM pressure, neglecting a uniform cosmic ray component, of $1.8 \times 10^4 \text{ K cm}^{-3}$. The gravitational energy of a cloud with radius a can be expressed in terms

of its observable mean surface density Σ or average hydrogen column density \bar{N}_H as [21]

$$-(W/3V) = (3\pi a/20)G\Sigma \simeq (1.39 \times 10^5 \text{ K cm}^{-3})(\bar{N}_H/10^{22} \text{ cm}^{-2}). \quad (5)$$

For the local dust-to-gas ratio, $\bar{N}_H = 10^{22} \text{ cm}^{-2}$ corresponds to a visual extinction of $A_V = 7.5$ mag. Since molecular clouds typically have $A_V \gg 2$, $P_{ext} \ll -W/3V$ and so, under the assumptions given, they are gravitationally bound.

3 Validity of Standard Assumptions

Are the assumptions that go into the standard argument still well-supported? Let's begin with the assumption that molecular clouds live for many free-fall times, and therefore have time to virialize. Blitz & Shu [8] estimated that cloud lifetimes were around 30 Myr in the Milky Way based on three main arguments: the ages of stars thought to be associated with the clouds; cloud locations downstream from dust lanes thought to be associated with cloud-forming spiral-arm shocks; and the comparison between the total molecular mass and the star-formation rate in the Galaxy. In the last several years, much shorter lifetimes have been proposed by Fukui et al. [11] based on the ages of stellar clusters associated with molecular gas; and by Ballesteros-Paredes et al. [3] based on the lack of a population of post-T Tauri stars with ages > 10 Myr closely associated with molecular gas. If molecular cloud lifetimes are only 2–3 τ_H , they may not live long enough to reach virial equilibrium.

Larson [18] found that the size of observed molecular clouds appears to correlate with the linewidth $R \propto \sigma^2$ and with the mean density of the cloud $R \propto \rho^{-1}$. These relations can be interpreted as evidence of virial equilibrium using the definition of M_{vir} given in equation (3), by noting that if they hold,

$$M_{vir} \propto R\sigma^2 \propto \rho R^3, \quad (6)$$

so the virial mass equals the actual mass of the cloud.

Ballesteros-Paredes et al. [6] pointed out that this actually only guarantees that kinetic and gravitational energy are equal, but not that full virial equilibrium holds, because of the extra terms neglected in equation (3). Even worse, both Vázquez-Semadeni et al. [26] and Ballesteros-Paredes & Mac Low [5] found that simulated observations of turbulent models appear to reproduce the density-size relationship, but that the actual density distribution did *not*. Figure 1 demonstrates this. Is even the energy equipartition argument merely an observational artifact?

On the other hand, the size-linewidth relationship appears in turbulent models ¹ even in the complete *absence* of self-gravity [25]. This can be understood as a direct consequence of the steep velocity power spectrum expected

¹ On a related subject, Brunt & Mac Low [10] examined the dispersion of velocity centroids in real and simulated observations. They found that density inho-

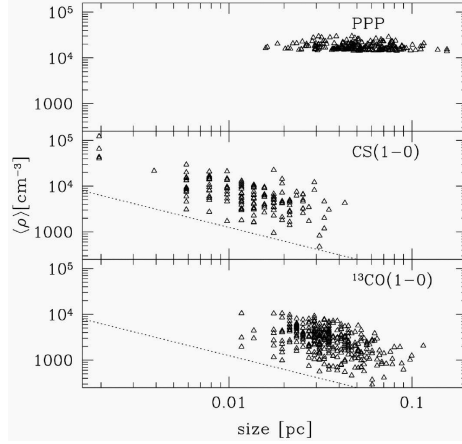


Fig. 1. Mean density-size relationship for clumps in a turbulent model measured in physical space (top) and in simulated observational coordinates for two different species (middle and bottom). The dotted line has a slope of -1 . In physical space we find no correlation, verifying the results by VBR [26], but nevertheless the simulated observations show such a correlation, as found by Larson (1981) and many others. The correlation in observational space may simply be the effect of the limited dynamic range of observations giving an effective column density cutoff.

in turbulence, with index between $-5/3$ and -2 . Larger objects have greater velocity differences, and thus larger velocity dispersion. However, this offers no support for the hypothesis that observed velocity dispersion are primarily due to self-gravity bringing clouds to a simple equilibrium between kinetic and gravitational energy.

In summary, the approximation of virial equilibrium for molecular clouds, especially in its simplest expression of equipartition, may be misleading. Their apparent short lifetimes suggest that the transient surface terms and distortions cannot be averaged away, and may dominate the dynamics of the cloud. Ballesteros-Paredes [6] measured the transient terms in turbulent simulations, and indeed found them to be substantial.

4 Clouds in Supernova-Driven Turbulence

In order to study the structure of density enhancements in a turbulent flow more carefully, we turn to a numerical model of supernova-driven turbulence [1, 2], done with an adaptive mesh refinement code and including: heating

mogeneity in hypersonic turbulence cancels projection smoothing, so that observed two-dimensional velocity centroid scaling actually gives the correct three-dimensional result, by lucky coincidence. This is not true in trans- or sub-sonic turbulence.

initially balancing cooling using an equilibrium ionization cooling curve; the galactic gravitational potential (but not self-gravity); and supernovae occurring randomly at the galactic rate. In this simulation, large density enhancements form and dissipate due to the turbulent compression and cooling, without the participation of self-gravity. The left side of Figure 2 shows that pressures in some dense regions reach values more than an order of magnitude higher than the average, as observed in molecular clouds. The pressure

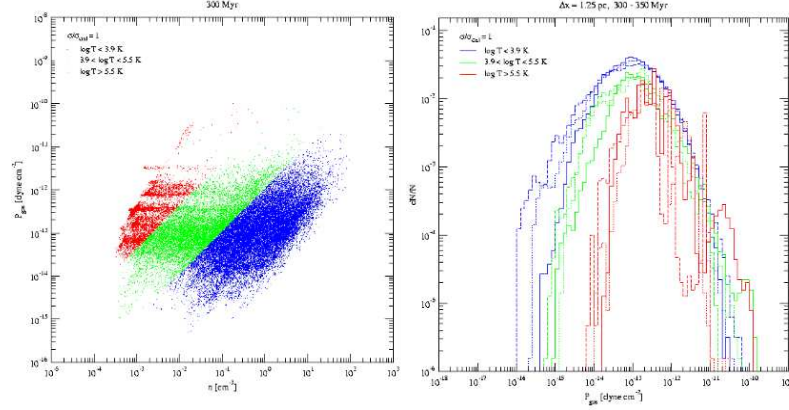


Fig. 2. *Left:* Scatter plot of pressure vs. density in the midplane of the SN-driven model showing occupation of high-pressure, high-density region associated with molecular clouds. *Right:* Volume-weighted probability distribution function of pressure in the same model. Note that the small volumes occupied by high-density, cold gas have large mass.

probability density function on the right side of the Figure also shows the broad distribution of pressures. The high-density, high-pressure regions are not fully resolved in these models, but they have sizes, shapes, and masses consistent with giant molecular clouds. They *must* be transient, probably dispersing on an internal crossing time [27], as they cannot be in equilibrium in the absence of self-gravity.

If molecular clouds are generally transient, ram-pressure confined objects, then why do CO luminosity, dust extinction, and other observational measures of cloud mass correlate so well with apparent virial mass (see Walter in these proceedings for example)? As we noted above, a turbulent flow produces a size-linewidth relation $R \propto \sigma^2$. The usual expression for the virial mass, equation (3), can thus be rewritten as $M \propto R^2$. Any collection of clouds whose mass is proportional to the square rather than the cube of their typical size will thus appear to be “virialized”. Ram pressure confined objects will usually be sheetlike, rather than spheres, and will thus naturally have $M \propto R^2$ rather than $M \propto R^3$. The filamentary shapes of observed clouds is

also consistent with this idea. Apparent virialization of clouds may actually just be a geometrical effect produced by a compressible turbulent flow.

5 Hydrostatic Equilibrium

The assumption of hydrostatic equilibrium between a turbulent pressure and gravity can also be misleading, because ram pressure is not isotropic. Numerical models of self-gravitating, turbulent gas reveal unexpected behavior. In the absence of continuing energy input, turbulence decays quickly [20], and collapse proceeds without substantial delay [15], whether or not fields are present [7] (so long as they cannot support magnetostatically).

Even driven turbulence strong enough that the turbulent pressure satisfies the Jeans criterion for stability against gravitational collapse does not completely prevent local collapse [17]. Adding weak magnetic fields reduces the amount of collapse, but does not prevent it entirely [12], a result that has now been confirmed at resolutions ranging from 64^3 zones to 512^3 zones by Li et al. [19], as shown in Figure 3. Collapse occurs if the mass in a region exceeds the *local* Jeans mass $m_{J,T} \propto v^3 \rho^{-1/2}$. Compressible turbulence produces density enhancements with $\Delta\rho/\rho \propto v^2/c_s^2$, so even if a region is *globally* supported by supersonic turbulence, *local* regions may still become gravitationally unstable [17]. These results suggest that most observed cores are dynamically collapsing.

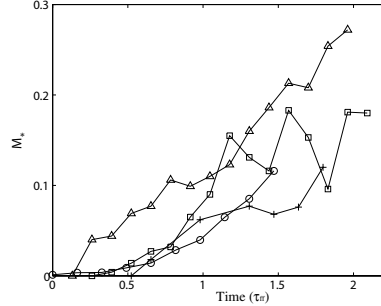


Fig. 3. Comparison of the mass accretion behavior for runs driven at $k = 1$ – 2 with varying resolution 64^3 (triangle), 128^3 (square), 256^3 (plus), and 512^3 (circle) [12, 19]. M_* denotes the sum of masses found in all cores, in units of box mass, determined by the modified CLUMPFIND [28]. Collapse rates vary, but collapse occurs in all cases, with the rate of collapse generally converging in the higher resolution models.

Observations that appear to suggest cores in hydrostatic equilibrium also must be interpreted with great care. More than 50% of the cores that appear

in a simulated observation of a numerical model of supersonic, hydrodynamic turbulence *without* self-gravity can be fit well by a projected Bonnor-Ebert sphere [4], as shown in Figure 4.

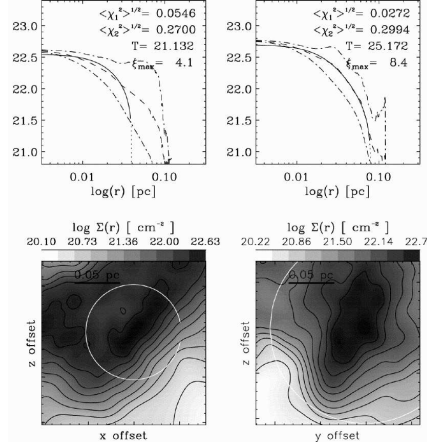


Fig. 4. Column density maps and radial profiles for xy and xz projections of a sample clump from an SPH simulation. The white circles show in each case the size of the radius used in the Bonnor-Ebert fit. Note the different morphology that the same core shows in the different projections [4].

Gravity clearly does dominate some regions in molecular clouds. However, gravitational collapse seems to proceed efficiently there, probably leading to more than half the mass forming stars [16, 17].

Large-scale gravitational instability forms regions with masses of order the thermal Jeans mass of the diffuse ISM $M_J > 10^7 M_\odot$ [13, 14]. These instabilities do not directly form single molecular clouds, but they may form regions of active star formation in which large molecular clouds form incidentally during gravitational collapse and fragmentation of the larger region.

6 Conclusions, or Questions

Are most or all observed clouds out of equilibrium? Are they either ram pressure confined and transient, *or* gravitationally collapsing on a free-fall time? The observational distinction between these two states may be whether associated star formation is occurring on a scale much longer than or close to the free-fall time. Is apparently bimodal star formation actually a continuum from low to high turbulent support? Finally, do clouds appear to be in virial equilibrium with $M \propto R\sigma^2$ because they are actually sheet-like objects in a turbulent flow with $M \propto R^2$ and $R \propto \sigma^2$?

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